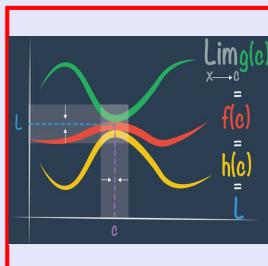


Math 261
Spring 2023
Lecture 30



Feb 19-8:47 AM

Graphing Functions using Calculus

when $f'(x) = 0$ or undefined,
 we have Critical numbers.

when $f'(x) > 0 \Rightarrow$ Increasing function

when $f'(x) < 0 \Rightarrow$ Decreasing function

when $f''(x) = 0$ or undefined,
 we have a possible inflection point.

when $f''(x) > 0 \Rightarrow$ Concave up

when $f''(x) < 0 \Rightarrow$ Concave down

I require a chart to combine all
 these informations.

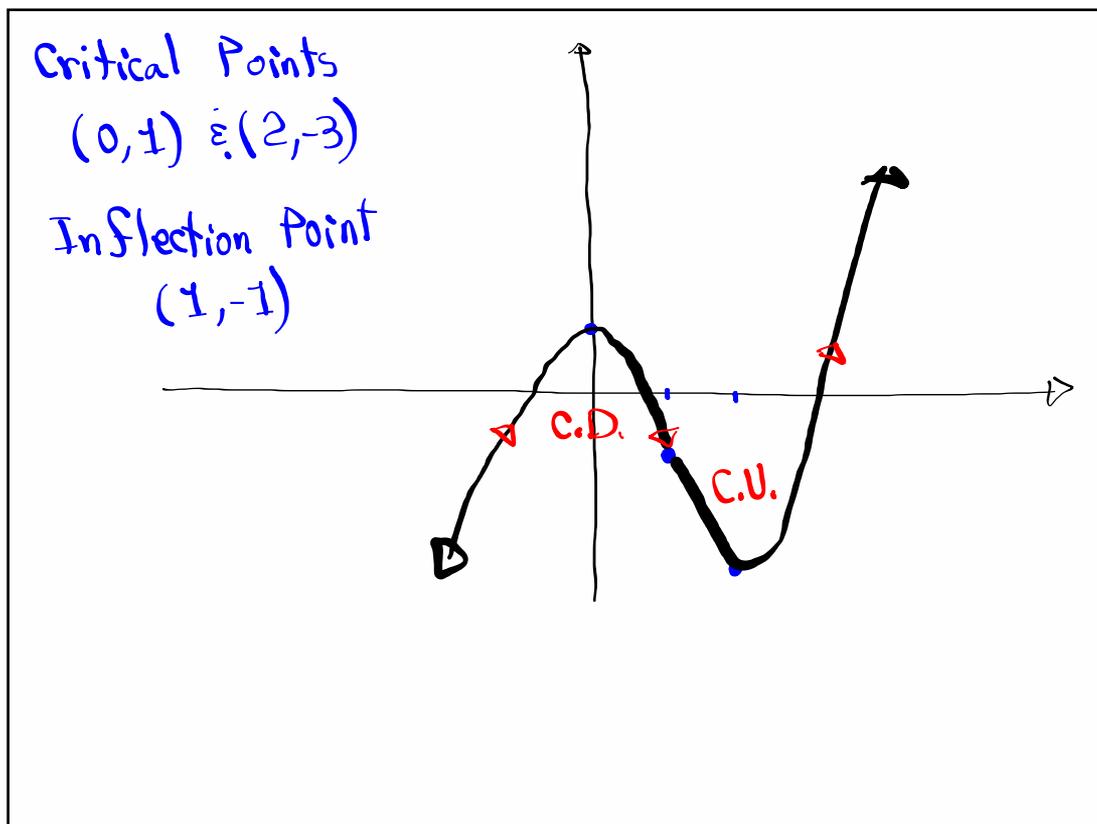
Apr 10-8:48 AM

$f(x) = x^3 - 3x^2 + 1$
 Polynomial \rightarrow Cont. everywhere $(-\infty, \infty)$
 $f'(x) = 3x^2 - 6x$ $f''(x) = 6x - 6$
 $f'(x) = 3x(x - 2)$ $f''(x) = 6(x - 1)$
 $f'(x) = 0 \Rightarrow x = 0, x = 2$ $f''(x) = 0 \Rightarrow x = 1$

x	$-\infty$	0	1	2	∞
$f'(x)$	+	•	-	•	+
$f''(x)$	-	-	•	+	+
$f(x)$					

$f(0) = 0^3 - 3(0)^2 + 1 = 1$ $f(1) = 1^3 - 3(1)^2 + 1 = -1$ $f(2) = 2^3 - 3(2)^2 + 1 = -3$

Apr 10-8:53 AM



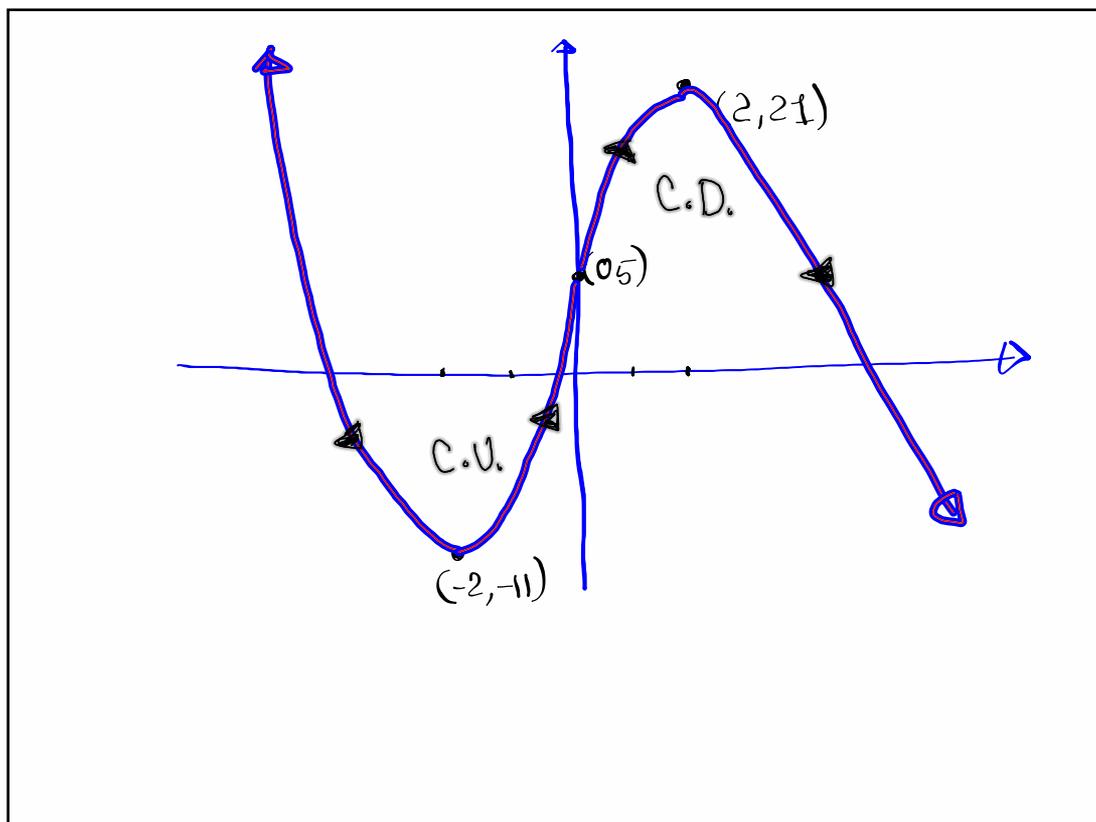
Apr 10-9:03 AM

$f(x) = -x^3 + 12x + 5$
 Polynomial \rightarrow Cont. everywhere $(-\infty, \infty)$
 Y-Int $(0, f(0)) = (0, 5)$
 $f'(x) = -3x^2 + 12$ $f''(x) = -6x$
 $f'(x) = -3(x^2 - 4)$ $f''(x) = 0 \rightarrow x = 0$
 $f'(x) = -3(x+2)(x-2)$
 $f'(x) = 0 \rightarrow x = -2, x = 2$

x	$-\infty$	-2	0	2	∞
$f'(x)$	-	•	+	•	-
$f''(x)$	+	+	•	-	-
$f(x)$					

$f(-2) = -(-2)^3 + 12(-2) + 5 = +8 - 24 + 5 = \boxed{-11}$
 $f(0) = -0^3 + 12(0) + 5 = \boxed{5}$
 $f(2) = -(2)^3 + 12(2) + 5 = -8 + 24 + 5 = \boxed{21}$

Apr 10-9:06 AM



Apr 10-9:17 AM

Suppose $f(x)$ & $g(x)$ are both positive and increasing functions, what about $(f \cdot g)(x)$?

$$\text{Let } h(x) = f(x) \cdot g(x)$$

$$h'(x) = \underbrace{f'(x)}_+ \cdot \underbrace{g(x)}_+ + \underbrace{f(x)}_+ \cdot \underbrace{g'(x)}_+$$

$$\text{So } h'(x) > 0$$

$h(x)$ must be an increasing function.

Apr 10-9:21 AM

Given $f(x) = x^3 - 3x + 2$

Polynomial \rightarrow Cont. everywhere $(-\infty, \infty)$

Y-Int $\rightarrow (0, f(0)) = (0, 2)$

Is 1 a solution for $f(x) = 0$?

Do we have x-Int. at $x=1$?

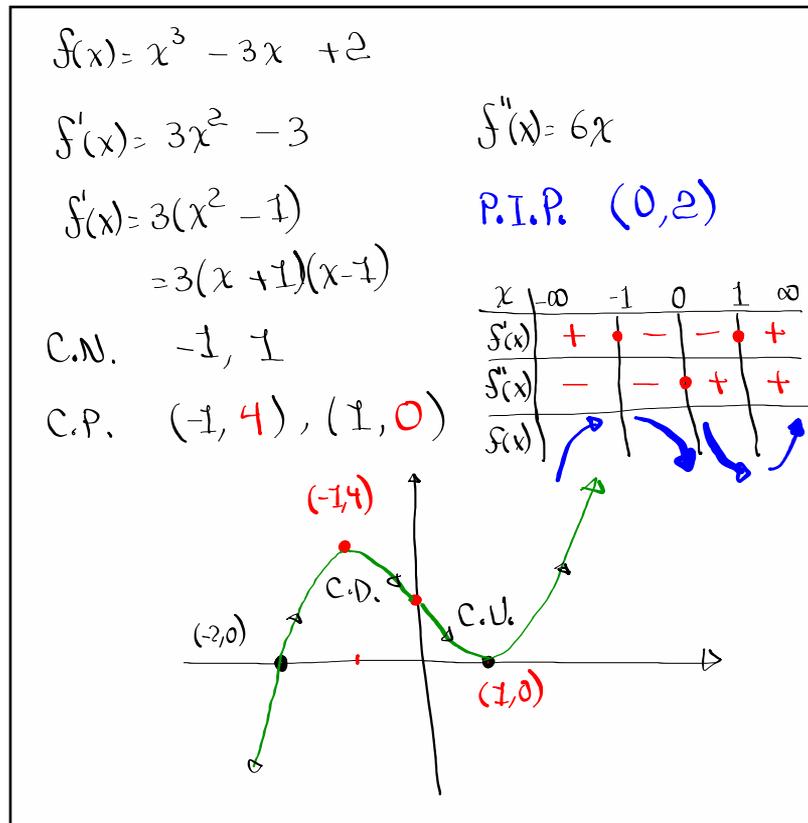
1	1	0	-3	2	Yes
		1	1	-2	
1	1	-2	0		x-Int (1,0)

$$f(x) = (x-1)(x^2 + x - 2) = (x-1)(x+2)(x-1)$$

we have the second
x-Int $\rightarrow (-2, 0)$

x-Int (1,0) is a repeated x-Int.
(twice)

Apr 10-9:26 AM



Apr 10-9:32 AM

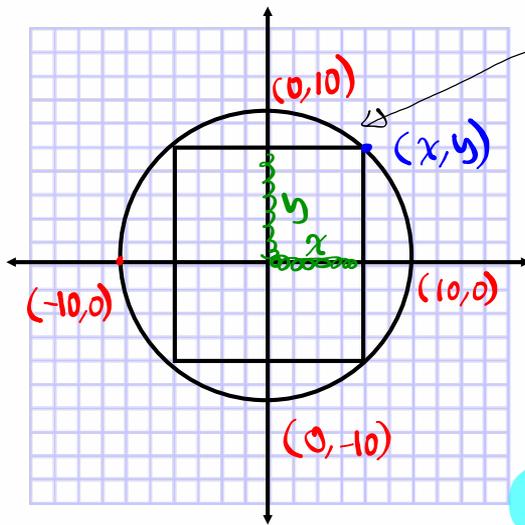
Find two numbers with sum of 10 and their product is as large as possible.

Two numbers	Product	x & y are these two numbers
0, 10	0	$x + y = 10$
1, 9	9	Product $x \cdot y$
\vdots		$x(10-x)$ to be maximum.
4, 6	24	
5, 5	25	

$f(x) = x(10-x)$
 $= 10x - x^2$ ← Parabola, opens down
 $f'(x) = 10 - 2x$ $10 - 2x = 0$ $x = 5$
 $f''(x) = -2$ → C.D.

Apr 10-9:42 AM

Consider a Circle with radius 10.



$$x^2 + y^2 = 100 \begin{cases} \rightarrow y^2 = 100 - x^2 \\ \rightarrow y = \sqrt{100 - x^2} \end{cases}$$

This rectangle to have **max. area.**

$$\text{Area} = 4xy$$

$$\text{Area} = 4x \cdot \sqrt{100 - x^2}$$

$$\downarrow$$

$$f(x) = 4x(100 - x^2)^{1/2}$$

Apr 10-9:50 AM